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CONVECTIVE OVERSHOOTING IN THE EVOLUTION OF VERY MASSIVE STARS

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ABSTRACT

Two important instances of possible convective overshooting in stars of 30–120 $M_{\odot}$ are considered here: (1) the possible merger between the convective core and the fully convective intermediate zone in the envelope at the end of the main phase of core hydrogen burning and (2) the possible penetration by the outer convection zone into the hydrogen-shell region when the star is a red supergiant. In the first instance, convective mixing between the core and inner envelope leads to renewed hydrogen burning near the center and, consequently, to a widening of the main-sequence band in the H-R diagram. In the second instance, deep penetration by the outer convection zone forces the star out of the red-supergiant configuration and into a new configuration very near the main sequence. It would thus seem possible to account for the apparent spread of the uppermost part of the observed main sequence and for the observed concentration of very luminous supergiants toward earlier spectral types, without having to postulate the occurrence of heavy mass loss.

Subject headings: convection — stars: interiors — stars: massive

I. INTRODUCTION

The extent of overshooting by convection cells from turbulent layers into stable radiative layers in the deep interior of stars is not known very accurately at present, but may be significant for stellar evolution if widespread mixing should happen to occur. In the case of unevolved stars on the upper main sequence, estimates of the overshoot distance d from the formal boundary of the convective core (the layer at which $\nabla_{\rm rad} = \nabla_{\rm ad}$) have been made by a number of authors, whose results can be expressed in terms of the local pressure scale height H_P. Mixing length theory, in the local approximation, suggests that $d \approx 10^{-2}\, H_P$ (Saslaw and Schwarzschild 1965; Roxburgh 1965), which is clearly negligible, but more accurate, nonlocal theories predict values of d/H_P equal to 0.07 (Shaviv and Salpeter 1973), 0.15 (Maeder 1975; Maeder and Bouvier 1976), 0.23 (Cogan 1975), $\sim$0.7 (Cloutman 1978; Cloutman and Whitaker 1980), and 0.27–0.71 (Roxburgh 1978; for details of the stellar models used by Roxburgh, see Härm and Rogerson 1955). The effect of stellar rotation on convective overshooting has also been investigated (Huppert and Spiegel 1977); for partially evolved stellar models in uniform rotation at breakup speed, we can estimate a value of $d/H_P \approx 10^{-2}$ at the convective core boundary by using Huppert and Spiegel's equations. These different results suggest that the correct magnitude of d/H_P could be anywhere between 10^{-2} and 0.7—and possibly even larger (Shaviv and Salpeter 1973; Maeder 1975). Of course, d/H_P cannot be an exact constant, as it would be in simple mixing length theory, because real convection is not fully describable by such a crude theory, which in fact was not used in the more elaborate calculations of Roxburgh (1978) and of Cloutman and Whitaker (1980). Whether the effect of overshooting is ultimately manifested as semiconvection or simply as an enlarged convective core will depend on the size of d as well as on the choking influence of the gradient of mean molecular weight at the core boundary.

A second aspect of convective overshooting inside stars applies only to very massive stars, provided that their structure can otherwise be understood with standard assumptions. When hydrogen becomes exhausted at the center of these stars, a large fully convective zone (FCZ) forms rapidly at the bottom of the hydrogen envelope. If the Schwarzschild (rather than the Ledoux) criterion for convection is adopted, the base of the FCZ penetrates close to the convective core. In stellar models having masses greater than 30 $M_{\odot}$, the minimum separation of the two convective regions is less than 1 pressure (or density) scale height (Stothers and Chin 1976, 1979). It is therefore possible that convective overshooting may connect and merge the two regions. Overshooting may be aided by meridional circulation if the stars are rotating, because the normally strong barrier against rotational mixing currents that is set up by a gradient of mean molecular weight μ (Mestel 1953, 1957; Kippenhahn 1974; Huppert and Spiegel 1977) has in the present instance an extremely small width, being formed wholly by the leading edge of the inward-growing convection zone. On the other hand, such a steep μ-gradient will tend to prevent the very overshoot of the convection cells that established the μ-gradient in the first place. Allowance for some overshoot, however, is always implicitly made by assigning the lower boundary of the convective zone to the layer where the condition $\nabla_{\rm rad} = \nabla_{\rm ad}$ is met on the *upper* side of the μ-discontinuity (see, e.g., Unno 1965; Ziółkowski 1972; Paczyński 1977). Further overshoot is certainly possible: arguments given by Böhm (1963) for the solar convection zone suggest that the overshoot distance could be a significant fraction of 1 pressure scale height. Perhaps in the present context overshoot would appear as semiconvection (Stothers and Chin 1976). If the two convection zones do merge, the consequences of a prolonged phase of core hydrogen burning may be observable.

There is a third situation where convective overshooting may be important. If the star becomes a red supergiant after the onset of core helium burning, the bottom of the outer convection zone may eventually penetrate into the hydrogen-deficient layers. Then a situation somewhat similar to the one just discussed will arise. However, three differences are to be noted. First, the hydrogen-deficient layers in the present instance are not totally devoid of hydrogen, and the μ-gradient through them will tend to oppose convection. Second, the metals opacity at the base of the outer convection zone may not be completely negligible; its contribution to the total opacity would tend, other things being equal, to deepen the convective envelope by steepening the local radiative temperature gradient. Third, convection near the surface is strongly nonadiabatic, so that the depth of the convective envelope sometimes depends critically on what choices are made for the convective mixing length and for the atmospheric opacities. If the base of the convective envelope penetrates close to the hydrogen-burning shell, the flattening of the hydrogen profile through the envelope can cause the star to evolve out of the red-supergiant configuration (Stothers and Chin 1968, 1973). It is important to know precisely when such a transition will take place, if one is to be able to predict the relative numbers of red and blue supergiants that ought to be observed.

The foregoing list does not exhaust the possibilities for convective overshooting in stars, but does contain those situations that are relevant to the evolution of the most massive stars before the end of core helium burning. The evolution of very massive stars with different amounts of hydrogen-core overshooting has already been studied in several papers (Stothers 1970, 1972; Massevitch *et al.* 1979; Cloutman and Whitaker 1980). In the present paper, we wish to investigate the consequences of assuming extensive overshooting from (1) the FCZ and (2) the outer convection zone.

II. OVERSHOOTING BETWEEN THE FCZ AND THE CONVECTIVE CORE

To lay out more quantitatively the prospects for significant overshooting between the FCZ and the convective core, we present Table 1, which lists the minimum separation in mass fraction (Δq) and in pressure ($|\Delta \ln P|$) that occurs between the two convectively unstable zones in models of massive stars. The stellar models used for this tabulation are taken from four standard evolutionary sequences, based on the following assumptions: an initial (hydrogen, metals) content of $(X_e, Z_e) = (0.739, 0.021)$; Cox-Stewart opacities; the Schwarzschild criterion for convection; and, of course, no convective overshooting. (If the Ledoux criterion is adopted, one finds in all cases that $|\Delta \ln P| > 1.5$, which represents a safe margin for stability against significant overshooting.)

TABLE 1

CLOSEST APPROACH OF THE FCZ TO THE CONVECTIVE CORE

$M/M_\odot$	First Approach Δq	First Approach $\lvert\Delta \ln P\rvert$	Second Approach Δq	Second Approach $\lvert\Delta \ln P\rvert$
15	0.18	1.7	...	...
30	0.13	0.9	0.24	1.8
60	0.09	0.6	0.18	1.5
120	0	0	0.12	1.1

In the case of 120 $M_\odot$, the two convectively unstable zones are found to merge when the advancing intermediate zone overtakes the shrinking convective core just after central hydrogen exhaustion; no overshooting need be assumed (a preliminary report of this result appeared in Stothers and Chin 1979). On the other hand, at a relatively low stellar mass like 15 $M_\odot$, a convective merger is most unlikely, because $|\Delta \ln P|$ is very large. The cases of 30 and 60 $M_\odot$ require special consideration, as $|\Delta \ln P|$ drops to 0.9 for $\sim$ 300 yr at 30 $M_\odot$, and to 0.6 for $\sim$ 3000 yr at 60 $M_\odot$. Within the available time, convective overshooting could possibly be effective, in view of the fact that the turnover time for convection cells in the two unstable zones is only a few weeks. If overshooting from the two zones happens to be equally efficient in both directions, the minimum overshoot distance needed to merge the zones is $\sim 0.4 H_P$, which does not seem like an unrealistic requirement. Moreover, if the stars are rotating, Eddington-Vogt circulation currents within the radiative buffer region may help to connect the two convection zones. Using an approximate formula derived by Kippenhahn (1974), we estimate the rotational mixing time in the radiative region as $500/\chi$ yr for 30 $M_\odot$ and $300/\chi$ yr for 60 $M_\odot$, where χ stands for the ratio of centrifugal force to gravity at the equator. Rotation may therefore have some effect, even though its importance is not likely to be dominant. Even without these special considerations the two convection zones may well merge, because the radiative temperature gradient is only slightly less steep than the adiabatic one across the radiative buffer region, and any small perturbation could upset the marginal state of stability here. In fact, at 60 $M_\odot$, an alternative and mathematically self-consistent solution to the structure equations exists in which the whole intermediate region is formally semiconvective. Physically, this would probably lead to a full merger of the two convection zones.

To study the observational consequences of allowing the two convection zones to combine, we have recomputed evolutionary tracks for 30 and 60 $M_\odot$ under the assumption that the two zones merge when $|\Delta \ln P|$ becomes smallest. For simplicity, we have also assumed that overshooting from the convective core during the prior phase of core hydrogen burning can be treated in the customary way by the formation of an overlying semiconvective zone. We shall discuss later the alternative assumption that overshooting creates a large extension of the convective core.

During the merger of the FCZ and the convective core, all the layers located below the outer boundary of the FCZ are very rapidly mixed. Fresh hydrogen is swept into the exhausted central region, which eventually contains $X_c = 0.08$, 0.12, and 0.08 in the stellar models with respectively 30, 60, and 120 $M_\odot$. The physical structure of the models consists of an enlarged hydrogen-burning

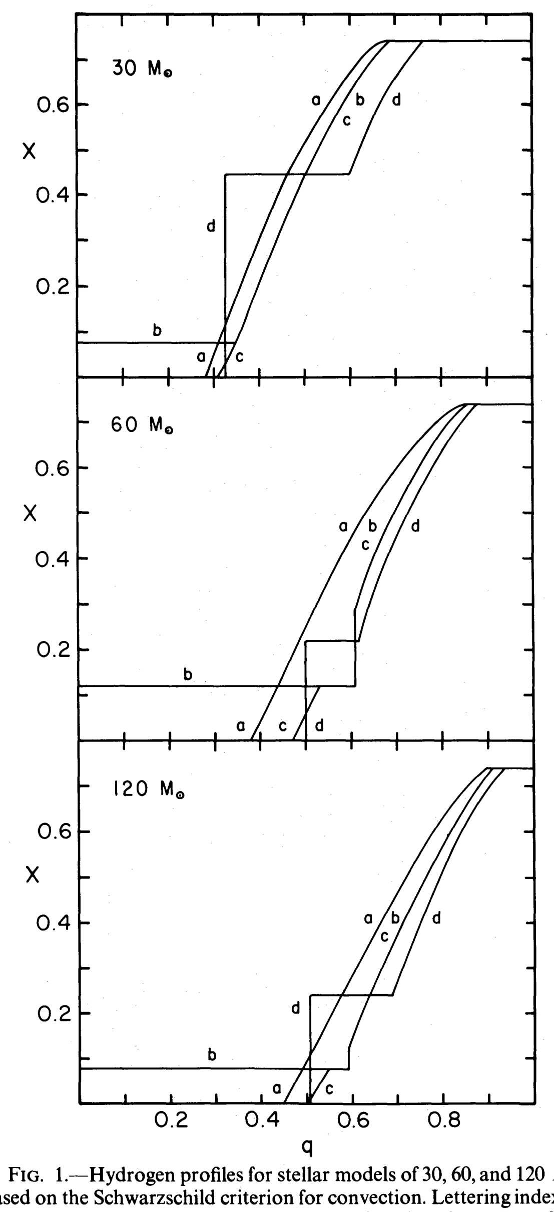

FIG. 1.—Hydrogen profiles for stellar models of 30, 60, and 120 $M_\odot$ based on the Schwarzschild criterion for convection. Lettering index: *a*, end of the original phase of core hydrogen burning; *b*, merger of the FCZ and the convective core; *c*, end of the second phase of core hydrogen burning; and *d*, maximum extent of the FCZ during core helium burning.

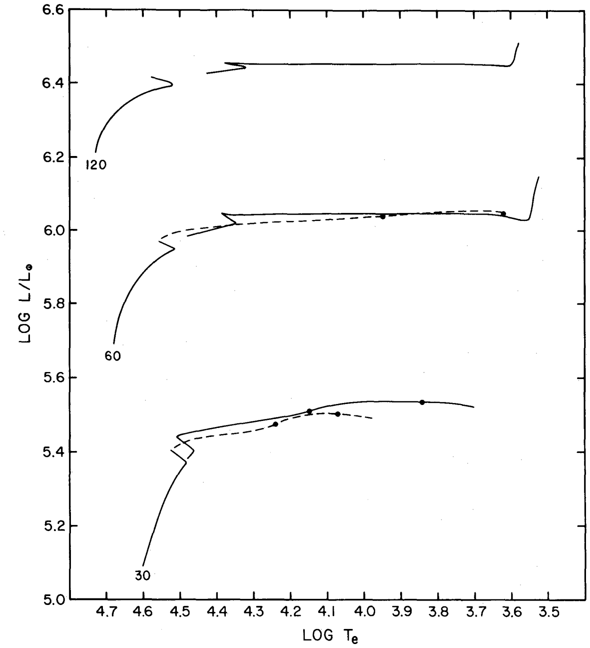

FIG. 2.—H-R diagram showing the evolutionary tracks for stellar models of 30, 60, and 120 $M_\odot$ based on the Schwarzschild criterion for convection. After the original phase of core hydrogen burning, evolution proceeds either with no merger between the FCZ and the convective core (*dashed lines*) or else with just such a merger (*solid lines*). Dots mark the beginning and end of the slow *blue* stages of core helium burning. All the tracks terminate at the stage of central helium exhaustion.

core, whose convective inner part lies well interior to the layers with a μ-gradient, except in the particular case of 30 $M_\odot$, where these layers remain semiconvective and attached to the convective core for a short while. Hydrogen profiles before and after the convective merger are shown in Figure 1.

The response that the star makes on the H-R diagram is an essentially discontinuous jump, as may be seen from the set of evolutionary tracks in Figure 2. The resulting gap between the star's previously coolest effective temperature and its new, even cooler state amounts to $\delta \log T_e = 0.001$, 0.034, or 0.090, if its mass is 30, 60, or 120 $M_\odot$, respectively. Unfortunately, such a small gap would not be easily observable. More interesting, because of its greater significance, is the further decrease of effective temperature caused by the star's evolution through a "second" phase of core hydrogen burning. One consequence of this temporary rejuvenation is that the main sequence becomes wider by $\delta \log T_e = 0.020$, 0.165, or 0.197 than in the absence of overshooting. Hydrogen-burning lifetimes are prolonged by 5%, 9%, or 6%. Luminosities are also somewhat brighter.

When hydrogen vanishes again at the center, a new FCZ develops at the bottom of the hydrogen-rich envelope. Now, however, the separation of the FCZ from the convective core is sufficiently large to render a merger very unlikely (see "second approach" entries in Table 1). In the stellar models with 30 $M_\odot$, the FCZ eventually grows to about the same size that it would have attained if the original merger had not taken place (compare Fig. 1 with Stothers and Chin 1976, Fig. 5). Consequently, the evolutionary track on the H-R diagram shows relatively little change at 30 $M_\odot$ when overshooting is allowed for. The corresponding sequences at 60 and 120 $M_\odot$, on the other hand, form FCZs during their second phase of development which are so small and poor in hydrogen that they cannot prevent the star from evolving directly into a red supergiant. Unlike any other evolutionary sequence of which we are aware, the present sequences for 60 and 120 $M_\odot$ do not show a drop in luminosity immediately after the star becomes stable on the red-supergiant branch. In fact, the star continues to redden and to brighten throughout the entire phase of core helium burning.

A tabulation of the most important ranges of effective temperature and evolution times encountered in the present evolutionary sequences is given in Table 2. Notation follows the usage of our previous papers.

In the preceding calculations we have ignored the possibility of extensive overshooting from the convective core during the phase of core hydrogen burning. This omission may be serious if d/H_P is relatively large. In this case a fraction $\sim 0.5\ d/H_P$ of the initial mass of the radiative envelope would be absorbed into the initial convective core. As a result, the zone with a μ-gradient would ultimately lie that much farther away from the center than in a standard calculation. Cloutman and Whitaker's (1980) overshoot calculations for a stellar model of 15.57 $M_\odot$ showed that an FCZ never developed at this mass. However, such strong stability against convection is unlikely to exist at considerably higher masses, because the initial convective core in an extremely massive star already occupies a large fraction of the star's mass (55%, 71%, and 83% for stars of respectively 30, 60, and 120 $M_\odot$) and especially because the zone with a μ-gradient in such a star is always relatively more unstable toward convection than in less massive stars. To demonstrate this point, we have computed another evolutionary track for 60 $M_\odot$ under the extreme assumption that the zone with a μ-gradient lies so far from the center that its upper boundary meets the stellar surface. (Note that mass loss from an initially more massive star could produce the same kind of object.) Shortly after the stage of central hydrogen exhaustion, an FCZ develops. Its separation from the convective core, amounting to $|\Delta \ln P| \approx 1$, differs but little from the results of a standard calculation. Since the prior enlargement of the hydrogen-burning convective core has already shifted the star upward and redward in the H-R diagram, the merger of the FCZ and the convective core simply accentuates a trend already begun. Therefore the results shown in Figure 2 can be regarded as highly conservative.

III. OVERSHOOTING FROM THE OUTER CONVECTION ZONE

Another part of the star where downward penetration by convection cells may be more important than conventionally thought is the base of the outer convection zone. Such a zone always exists if the star becomes a red supergiant. It is a curious observed fact that very massive red supergiants are extremely few in number. We can conjecture that an outer convection zone which is deeper than expected may transform these stars quickly into blue supergiants. Such a rapid metamorphosis is definitely required if the stars should happen to become red *early* during core helium burning, when a significant fraction of their total lifetime still remains to them. This will be the case if a moderate amount of prior core overshooting (or prior mass loss from the surface) has taken place, or if the Ledoux criterion for convection is the correct one to adopt (at least for stellar masses up to $\sim 45\ M_\odot$), or perhaps if very large atomic opacities occur in the stellar envelope.

To test our conjecture, we have adopted a model with a mass of 30 $M_\odot$ that has evolved to the tip of the red-supergiant branch, along a standard evolutionary track based on the Ledoux criterion for convection (Stothers and Chin 1979). The ratio of convective mixing length to local pressure scale height, α_P, has been set to 1, but can easily be varied. In standard tracks for 30 $M_\odot$ with a variety of convective mixing lengths, the stellar models may or may not leave the red-supergiant configuration during core helium burning. If they do, their departure seems always to take place quite late, when the hydrogen shell has burned very close to the chemical discontinuity that marks the limit of previous invasions by

TABLE 2

EVOLUTIONARY SEQUENCES FOR STARS OF 30, 60, AND 120 $M_\odot$ WITH AND WITHOUT OVERSHOOTING BETWEEN THE FCZ AND THE CONVECTIVE CORE[a]

$M/M_\odot$	Convective Overshooting?	log T_e (ZAMS)	log T_e (TAMS)	log T_e (tip)	log T_e (b/y)	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_{He}	τ_y/τ_{He}
30	no	4.60	4.49	4.24	~4.07	6.101	0.086	0.968	0.032
	yes	4.60	4.47	4.14	~3.85	6.389	0.074	0.973	0.027
60	no	4.68	4.52	3.94	~3.63	3.830	0.086	1.000	...
	yes	4.68	4.35	...	...	4.175	0.069	0.028	0.001
120	yes[b]	4.73	4.32	...	...	3.102	0.090	0.027	0.001

[a] $(X_e, Z_e) = (0.739, 0.021)$; $\alpha_P = 1$; Schwarzschild criterion for convection.
[b] Central hydrogen vanishes for the first time when $\tau = 2.919 \times 10^6$ yr; the corresponding phase has log T_e (TAMS) = 4.52.

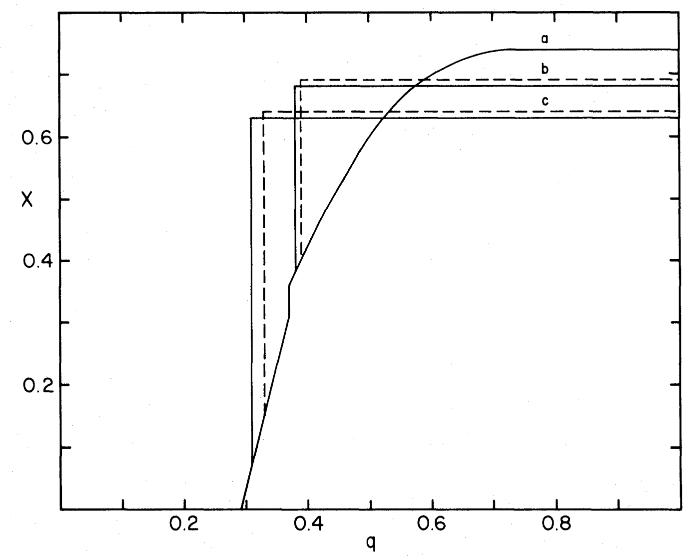

FIG. 3.—Hydrogen profiles for stellar models of 30 $M_\odot$ based on the Ledoux criterion for convection. Lettering index: *a*, initial evolutionary stage on the red-supergiant branch; *b*, tip of the red-supergiant branch in standard evolutionary tracks; and *c*, tip of the red-supergiant branch in evolutionary tracks that assume deep penetration by the outer convection zone. The ratio of convective mixing length to local pressure scale height has been taken to be either $\alpha_P = 1$ (*solid lines*) or $\alpha_P = 10$ (*dashed lines*).

the outer convection zone (Ziółkowski 1972; Stothers and Chin 1975, 1979).

To obtain the same general shape of hydrogen profile in the envelope at an earlier stage of evolution, we assume that, owing to one or more of the physical effects mentioned in § I, the outer convection zone initially penetrates very deeply into the hydrogen-deficient layers. A plausible assumption is that deepest penetration takes place, as it does normally, when the star lies at the tip of the red-supergiant branch. (Although this process of inward penetration occurs by means of full convection, it can be called "overshooting" in a loose sense.)

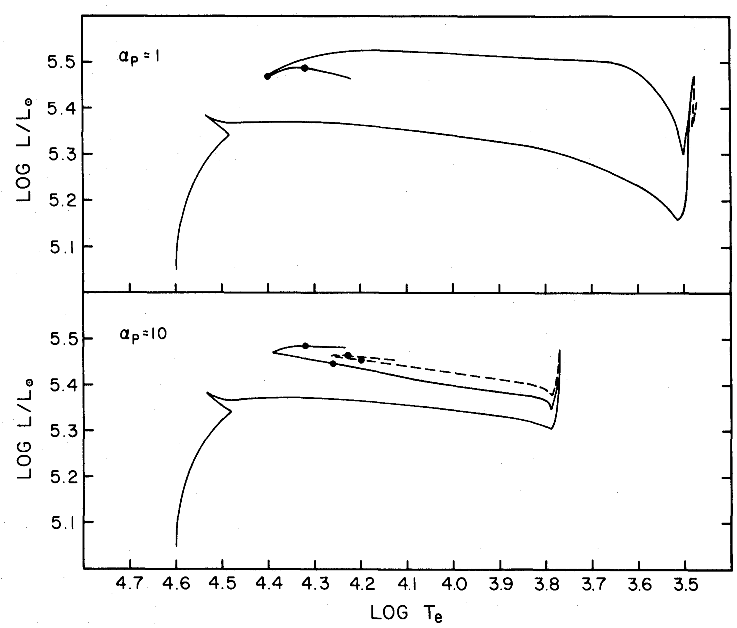

FIG. 4.—H-R diagram showing the evolutionary tracks for stellar models of 30 $M_\odot$ based on the Ledoux criterion for convection. Starting at the tip of the red-supergiant branch, evolution proceeds either with standard convective penetration (*dashed lines*) or else with very deep convective penetration (*solid lines*). Dots mark the beginning and end of the slow *blue* stages of core helium burning. All the tracks terminate at the stage of central helium exhaustion.

TABLE 3

EVOLUTIONARY SEQUENCES FOR STARS OF 30 $M_\odot$ WITH AND WITHOUT VERY DEEP PENETRATION BY THE OUTER CONVECTION ZONE[a]

α_P	Convective Overshooting?	log T_e (ZAMS)	log T_e (TAMS)	log T_e (tip)	log T_e (b/y)	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_{He}	τ_y/τ_{He}
1	no	4.60	4.49	...	...	5.763	0.082	0.034	0.005
	yes	4.60	4.49	4.40	~4.32	5.763	0.085	0.937	0.052
10	no	4.60	4.49	4.26	~4.20	5.763	0.082	0.426	0.057
	yes	4.60	4.49	4.39	~4.26	5.763	0.083	0.956	0.030

[a] $(X_e, Z_e) = (0.739, 0.021)$; Ledoux criterion for convection.

Computationally, our procedure will consist of the following steps: (1) selecting a mass fraction q_a, above which convective mixing is assumed to have homogenized the chemical composition; (2) relaxing the stellar model to its new hydrogen profile; and (3) evolving the model forward in time to see if and when a transition to a blue-supergiant configuration occurs. If such a transition fails to develop almost immediately, we must lower the value of q_a in the starting model and begin the whole process over again.

In this way, we have obtained the results shown in Figures 3 and 4 for the two cases $\alpha_P = 1$ and $\alpha_P = 10$. The very large value of $\alpha_P = 10$ was chosen both as an extreme case and as a device to obtain a blue loop even in the absence of "overshooting" (see Stothers and Chin 1979). The time that the star spends at effective temperatures lower than log $T_e = 3.9$ (this particular value is not critical) can be expressed as a percentage of the star's total helium-burning lifetime (which is about 4.8×10^5 yr in both "overshooting" cases). Up to the tip of the red-supergiant branch, the star consumes only 0.4% or 0.2% of its available time, if $\alpha_P = 1$ or 10, respectively; descending from the tip, it uses up another 0.7% or 1.2%. Thus the total time spent by the star in the vicinity of the red-supergiant branch is only slightly more than 1% in the "overshooting" cases. Statistically speaking, red supergiants that are this short-lived will not be observed. A larger predicted number, of course, could be obtained by choosing a larger value of q_a. But our object has been merely to show that it is possible to evolve massive stars *quickly* into and out of the red-supergiant configuration during the phase of core helium burning, without the necessity of invoking extensive mass loss (cf. Chiosi, Nasi, and Sreenivasan 1978; Stothers and Chin 1979). Only if the envelope opacities were very high would it be necessary to assume very heavy mass loss (Stothers and Chin 1977, 1978).

Quitting the region of red supergiants, the star rapidly crosses the H-R diagram and settles down in a narrow zone of high effective temperatures, where it completes the phase of core helium burning. Here a very close approach to the main sequence takes place, at about the time when the hydrogen-burning shell first makes full contact with the chemical discontinuity in the envelope. The highest effective temperature achieved during this phase depends almost entirely on the choice of q_a (see also Stothers and Chin 1968). It is worth mentioning that convection breaks out early and lingers long in the layers just above the hydrogen discontinuity; however, this region remains separated by at least 5 pressure scale heights from the convective core.

After helium is exhausted at the center, neutrino emission is expected to reduce the remaining lifetime to an unobservably small value. Vital data concerning the present evolutionary sequences are given in Table 3.

IV. CONCLUSION

Two areas in which convective overshooting may be important in the evolution of very massive stars ($M \geq 30\ M_\odot$) have been studied in the present paper. One area is concerned with the possible merger of the convective core and the fully convective intermediate zone that develops very suddenly in the envelope of the star at the end of the main phase of core hydrogen burning. If the two convective zones combine, there will be a brief renewal of hydrogen burning in the core. One observable consequence will be a moderate widening of the main-sequence band in the H-R diagram for luminosities brighter than log $(L/L_\odot) = 5.5$.

The second area under investigation has been the possibility of very deep penetration by the outer convection zone, should the star happen to become a red supergiant during the onset of core helium burning. If chemical homogenization of the hydrogen envelope extends relatively more deeply in red supergiants with higher masses, then a star of very high mass might emerge quickly from its red-supergiant state. This could help to explain why so few red supergiants with log $(L/L_\odot) > 5.3$ are actually observed. Another interesting result is that, having become once again a blue supergiant, a very massive star will proceed to approach very closely to the main sequence in the H-R diagram. Consequently, the predicted distribution of blue stars in the upper part of the H-R diagram will have, in this picture, the appearance of being almost continuous and mostly concentrated to very high effective temperatures, although, as far as actual observations are concerned, the persistently large width of the main sequence at luminosities as low as log $(L/L_\odot) = 4.5$ remains to be explained.

It is noteworthy that heavy mass loss need not be assumed in order to achieve these points of agreement with observation. Since the empirical rates of mass loss are poorly known in any case, they may just as well be assumed, tentatively, to be unimportant from an evolutionary point of view. Until other fundamental physical processes like convection and opacity are better understood, the true evolution of very massive stars will undoubtedly remain elusive.

The helpful comments of an anonymous referee led us to clarify and extend the results of § II.

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